DYNAMICS

Momentum

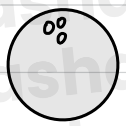
"It is a product of mass and velocity of a body."

momentum = mass x velocity

$\boxed{p = mv}$ SI Unit: Ns or $kgms^{-1}$

Vector

→ It tells about the amount/strength of motion of a body.



$m = 10\,kg$
$v = 2\,ms^{-1}$

$p = mv$
$p = 10 \times 2$
$p = 20\,Ns$

$m = 0.2\,kg$
$v = 200\,ms^{-1}$

$p = mv$
$p = 0.2 \times 200$
$p = 40\,Ns$

Change of momentum

change in momentum = final momentum − initial momentum

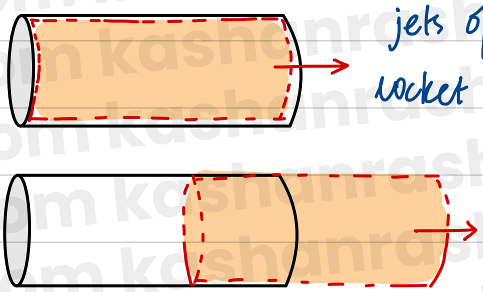
$$\boxed{\Delta p = p_f - p_i}$$

change in momentum

→ change in mass

→ change in velocity

change in mass



jets of liquids, rocket propulsion

$\Delta p = p_f - p_i$
$= m_f v - m_i v$
$= (m_f - m_i) v$

$$\boxed{\Delta p = \Delta m \times v}$$

change in velocity

speed up, slow down, direction changes

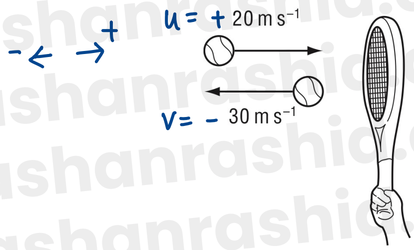
$\Delta p = p_f - p_i$
$= mv - mu$

$$\boxed{\Delta p = m(v-u)}$$ or

$\Delta p = m \times \Delta v$

v: final velocity u: initial vel.

9 A tennis ball of mass 100 g is struck by a tennis racket. The velocity of the ball is changed as shown.



$\Delta p = m(v - u)$

$\Delta p = 0.1(-30 - 20)$

$\Delta p = -5\,\text{Ns}$ or $5\,\text{kgms}^{-1}$

What is the magnitude of the change in momentum of the ball?

A $1\,\text{kg m s}^{-1}$ B $5\,\text{kg m s}^{-1}$ C $1000\,\text{kg m s}^{-1}$ D $5000\,\text{kg m s}^{-1}$

Force

"The rate of change of momentum."

$$\boxed{F_{net} = \frac{\Delta p}{\Delta t}}$$

Δp → change in momentum

Δt → time taken

F_{net} → resultant force

- SI Unit: Newton (N)
- Vector

• If $\Delta p = m(v-u)$, then

$$F_{net} = \frac{\Delta p}{\Delta t} = \frac{m(v-u)}{\Delta t}$$

$\frac{(v-u)}{\Delta t}$ → acc.

$$\boxed{F_{net} = ma}$$

• If $\Delta p = \Delta m \times v$, then

$$F_{net} = \frac{\Delta p}{\Delta t}$$

$$F_{net} = \frac{\Delta m \times v}{\Delta t}$$

$$\boxed{F_{net} = \dot{m} v}$$

$\dot{m} = \frac{\Delta m}{\Delta t}$

Δm → change in mass

Δt → time

$\dot{m}$ → mass flowrate

SI Unit: kgs^{-1}

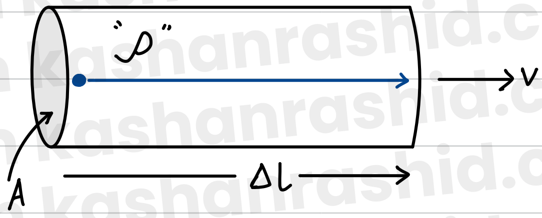
60 kg per second!

e.g. 60 kg of water passes through a pipe in 10 min. Determine the mass flowrate.

$$\dot{m} = \frac{\Delta m}{\Delta t} = \frac{60}{10 \times 60} = 0.1\,\text{kgs}^{-1}$$

$\dot{m} = \rho A v$ → velocity

ρ ↓ density, A ↓ cross-sectional area



$\dot{m} = \dfrac{\Delta m}{\Delta t}$

$\dot{m} = \dfrac{\rho \Delta V}{\Delta t}$ ↗ change in volume

$\dot{m} = \dfrac{\rho(A \times \Delta L)}{\Delta t}$ ↗ velocity

$$\boxed{\dot{m} = \rho A v}$$

$F_{net} = \dot{m} v$

$= (\rho A v) v$

$$\boxed{F_{net} = \rho A v^2}$$

Types of Collision

Elastic Collision	Inelastic Collision
1. Ideal collision	1. Real collision
2. No energy loss at heat.	2. Energy lost mainly as heat
3. Small masses e.g. atoms or molecules	3. Collision between large masses, explosion, bodies sticking after collision.
Momentum: Conserved	Momentum: Conserved
Kinetic Energy: Conserved	Kinetic Energy: Not conserved
Total Energy: Conserved	Total Energy: Conserved

Checking Elastic and Inelastic Collision

1. Kinetic energy

Total KE before collision = Total KE after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

If the above statement is valid, the collision is elastic.

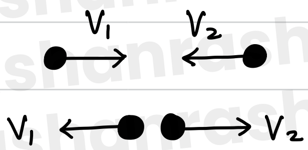
2. Relative Speed

relative speed of approach = relative speed of seperation
(before collision) (after collision)

If the above statement is valid, the collision is elastic.

same direction : subtract

rel speed = $V_2 - V_1 \,/\, V_1 - V_2$



opposite direction : add

rel speed = $V_1 + V_2$

"When two bodies of equal masses collide elastically, they exchange their velocities."

Law of Conservation of Momentum

"For an isolated system, the total momentum of bodies before collision is equal to their total momentum after collision."

before collision　　　　after collision

$$p_1 + p_2 = p_1 + p_2$$

$$\boxed{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2}$$ mostly used!

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$
$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$
$$\underbrace{-m_1(v_1 - u_1)}_{-\Delta p_1} = \underbrace{m_2(v_2 - u_2)}_{+\Delta p_2}$$

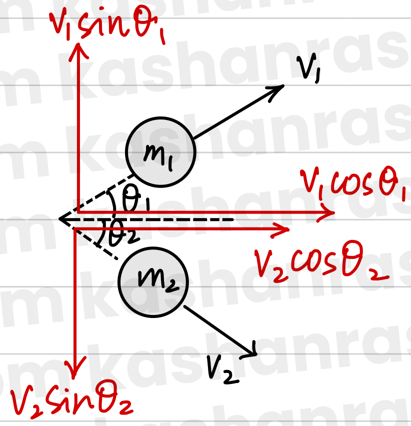
"For an isolated system, the momentum lost by one body is equal to that gained by the other."

$$-\Delta p_1 = \Delta p_2$$
$$\Delta p_1 + \Delta p_2 = 0$$
$$\Delta p_{system} = 0$$

"For an isolated system, the change in momentum of the entire system is zero."

2D-Momentum

before collision



after collision

Law of conservation of momentum will be applied along x-axis & y-axis seperately.

X-axis

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

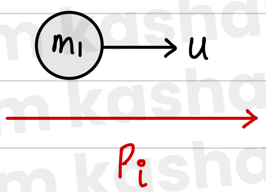
$$m_1 u_1 + m_2 u_2 = m_1 (v_1 \cos\theta_1) + m_2 (v_2 \cos\theta_2)$$

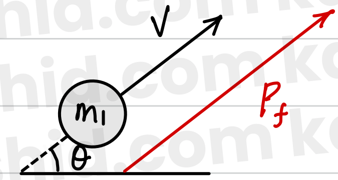
Y-axis

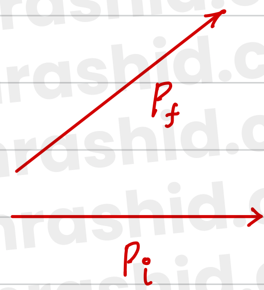
$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$0 + 0 = m_1 (v_1 \sin\theta_1) + m_2 (-v_2 \sin\theta_2)$$

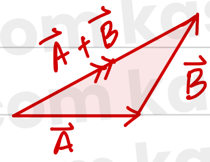
Change in Momentum in 2D



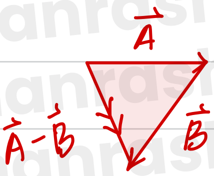




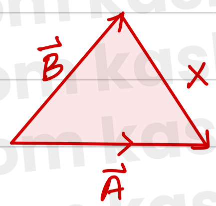
$A + B$



$A - B \simeq A + (-B)$

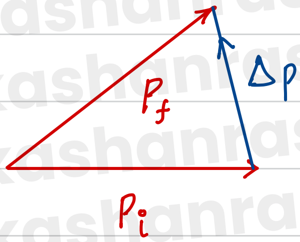


OR



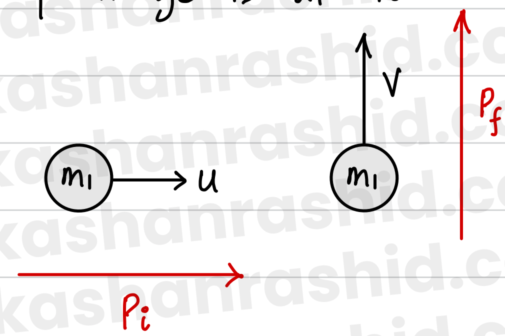
$B + X = A$

$X = A - B$

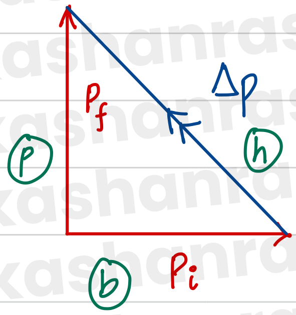


$\Delta p = P_f - P_i$ or $P_i + \Delta p = P_f$

if change is at 90°



$\Delta p = P_f - P_i$ or $P_i + \Delta p = P_f$



$h^2 = p^2 + b^2$

$\Delta p^2 = P_f^2 + P_i^2$

PHYSICS 9702/01

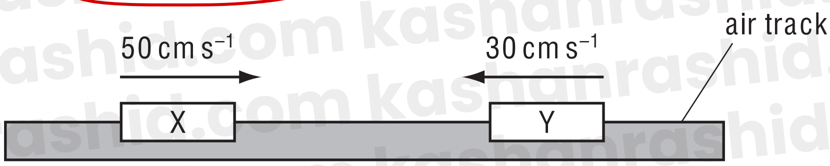
Paper 1 Multiple Choice May/June 2008

1 hour

Additional Materials:

Velocities exchange

10 Two equal masses X and Y are moving towards each other on a frictionless air track as shown. The masses make an elastic collision. DYNAMICS 2008



Which row gives possible velocities for the two masses after the collision?

	velocity of X	velocity of Y
A	zero	$20\,\text{cm s}^{-1}$ to the right
B	$10\,\text{cm s}^{-1}$ to the right	$10\,\text{cm s}^{-1}$ to the right
C	$20\,\text{cm s}^{-1}$ to the left	zero
D	$30\,\text{cm s}^{-1}$ to the left	$50\,\text{cm s}^{-1}$ to the right

PHYSICS 9702/01

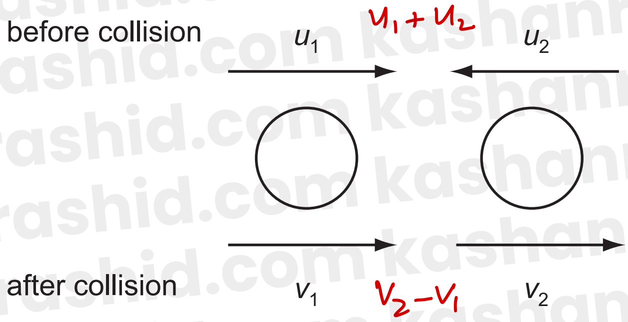
Paper 1 Multiple Choice October/November 2008

1 hour

Additional Materials:

10 Two spheres approach each other along the same straight line. Their speeds are u_1 and u_2 before collision, and v_1 and v_2 after collision, in the directions shown below.

$u_1 + u_2$



$v_2 - v_1$

Which equation is correct if the collision is perfectly elastic?

A $u_1 - u_2 = v_2 + v_1$

B $u_1 - u_2 = v_2 - v_1$

C $u_1 + u_2 = v_2 + v_1$

D $u_1 + u_2 = v_2 - v_1$

PHYSICS 9702/11

Paper 1 Multiple Choice May/June 2010

1 hour

Additional Materials:

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\cancel{m}(60) + \cancel{m}(-40) = (2\cancel{m})v$

$20 = 2v$

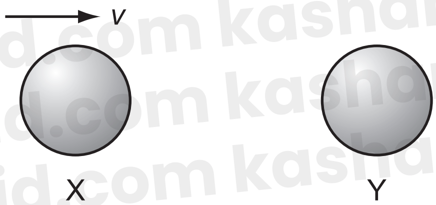
$v = 10\,ms^{-1}$

10 Two equal masses travel towards each other on a frictionless air track at speeds of 60 cm s^{-1} and 40 cm s^{-1}. They stick together on impact.

What is the speed of the masses after impact?

A 10 cm s^{-1} **B** 20 cm s^{-1} **C** 40 cm s^{-1} **D** 50 cm s^{-1}

12 The diagram shows two identical spheres X and Y.



Initially, X moves with speed v directly towards Y. Y is stationary. The spheres collide elastically.

What happens?

	X	Y
A	moves with speed $\frac{1}{2}v$ to the right	moves with speed $\frac{1}{2}v$ to the right
B	moves with speed v to the left	remains stationary
C	moves with speed $\frac{1}{2}v$ to the left	moves with speed $\frac{1}{2}v$ to the right
D	stops	moves with speed v to the right

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(20)(6) + (12)(-15) = (32)v$ / $20v + 12v$

$v = 1.88 \simeq 1.9\,ms^{-1}$

11 An object of mass 20 kg is travelling at a constant speed of $6.0\,m\,s^{-1}$.

It collides with an object of mass 12 kg travelling at a constant speed of $15\,m\,s^{-1}$ in the opposite direction. The objects stick together.

What is the speed of the objects immediately after the collision?

A $1.9\,m\,s^{-1}$ **B** $9.0\,m\,s^{-1}$ **C** $9.4\,m\,s^{-1}$ **D** $21\,m\,s^{-1}$

PHYSICS **9702/11**

Paper 1 Multiple Choice **May/June 2012**

1 hour

Additional Materials:

11 Two similar spheres, each of mass m and travelling with speed v, are moving towards each other.

The spheres have a head-on elastic collision.

Which statement is correct?

A The spheres stick together on impact.

B The total kinetic energy after impact is mv^2.

C The total kinetic energy before impact is zero.

D The total momentum before impact is $2mv$.

PHYSICS **9702/11**

Paper 1 Multiple Choice **May/June 2013**

1 hour

Additional Materials:

10 Which of the following is a statement of the principle of conservation of momentum?

- **A** In an elastic collision momentum is constant.
- **B** Momentum is the product of mass and velocity.
- **C** The force acting on a body is proportional to its rate of change of momentum.
- **D** The momentum of an isolated system is constant.

11 A 2.0 kg mass travelling at $3.0\,m\,s^{-1}$ on a frictionless surface collides head-on with a stationary 1.0 kg mass. The masses stick together on impact.

$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$

$(2)(3) + 0 = (3)V$

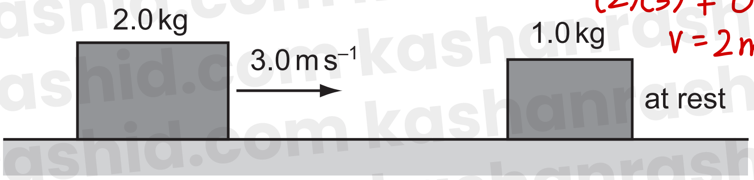
$V = 2\,ms^{-1}$

$\frac{1}{2}(2)(3)^2 + 0$

9J. – before

$\frac{1}{2}(3)(2)^2$

6J. – after



How much kinetic energy is lost on impact?

A zero **B** 2.0 J **C** 2.4 J **D** 3.0 J

PHYSICS **9702/12**

Paper 1 Multiple Choice **May/June 2013**

1 hour

Additional Materials:

10 Two bodies travelling in a straight line collide in a perfectly elastic collision. Which of the following statements **must** be correct?

- **A** The initial speed of one body will be the same as the final speed of the other body.
- **B** The relative speed of approach between the two bodies equals their relative speed of separation.
- **C** The total momentum is conserved but the total kinetic energy will be reduced.
- **D** One of the bodies will be stationary at one instant.

PHYSICS 9702/12

Paper 1 Multiple Choice May/June 2014

1 hour

Additional Materials:

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(5000)(2) + (5000)(-1) = (10000)v$

$v = 0.5\ ms^{-1}$

7 Two train carriages each of mass 5000 kg roll toward one another on a level track. One is travelling at $2.00\,\mathrm{m\,s^{-1}}$ and the other at $1.00\,\mathrm{m\,s^{-1}}$, as shown.

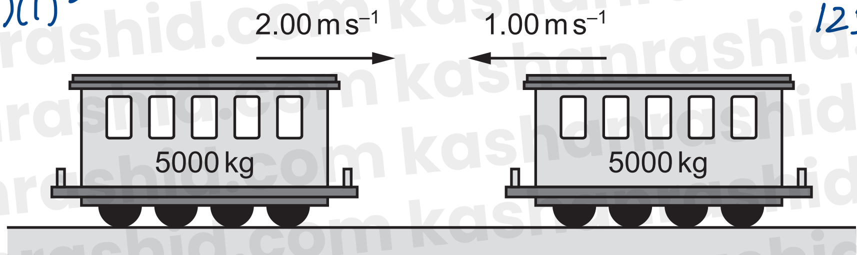
$\frac{1}{2}(5000)(2)^2 + \frac{1}{2}(5000)(1)^2$

$10{,}000 + 2500$

$12{,}500\ J$

$\frac{1}{2}(10{,}000)(0.5)^2$

$1250\ J$



$12500 - 1250$

$11250\ J$

They collide and join together.

What is the kinetic energy lost during the collision?

A 1250 J **B** 7500 J **C** 11 250 J **D** 12 500 J

PHYSICS 9702/13

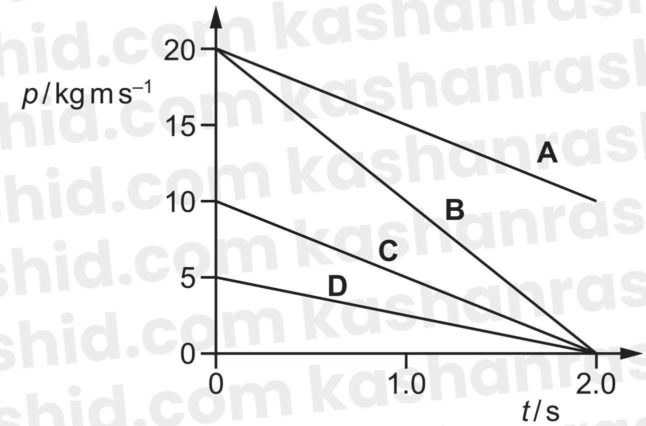
Paper 1 Multiple Choice May/June 2014

1 hour

Additional Materials:

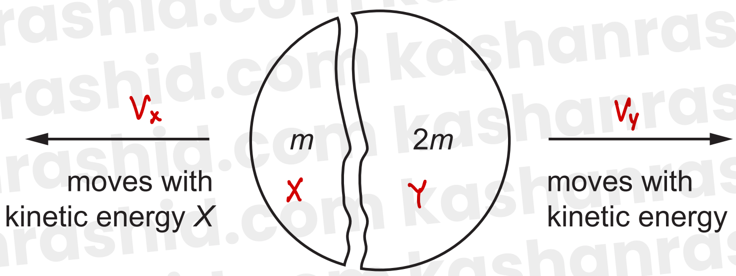
11 A resultant force of 10 N acts on a body for a time of 2.0 s.

Which graph could show the variation with time t of the momentum p of the body?



12 A stationary body explodes into two components of masses m and $2m$.

The components gain kinetic energies X and Y respectively.



What is the value of the ratio $\frac{X}{Y}$?

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** $\frac{2}{1}$ **D** $\frac{4}{1}$

$$\frac{X}{Y} = \frac{\cancel{\frac{1}{2}} m v^2}{\cancel{\frac{1}{2}} m v^2}$$

$$= \frac{\cancel{m} V_x^2}{2\cancel{m} V_y^2}$$

$$\frac{X}{Y} = \frac{V_x^2}{2V_y^2}$$

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

$$0 + 0 = m(-V_x) + (2m)V_y$$

$$V_x = 2V_y$$

$$\frac{X}{Y} = \frac{(2V_y)^2}{2V_y^2}$$

$$\frac{X}{Y} = \frac{4\cancel{V_y^2}}{2\cancel{V_y^2}}$$

hence $\boxed{\frac{X}{Y} = \frac{2}{1}}$

11 The diagram shows a man standing on a platform that is attached to a flexible pipe. Water is pumped through the pipe so that the man and platform remain at a constant height.

$F = \dot{m}v$

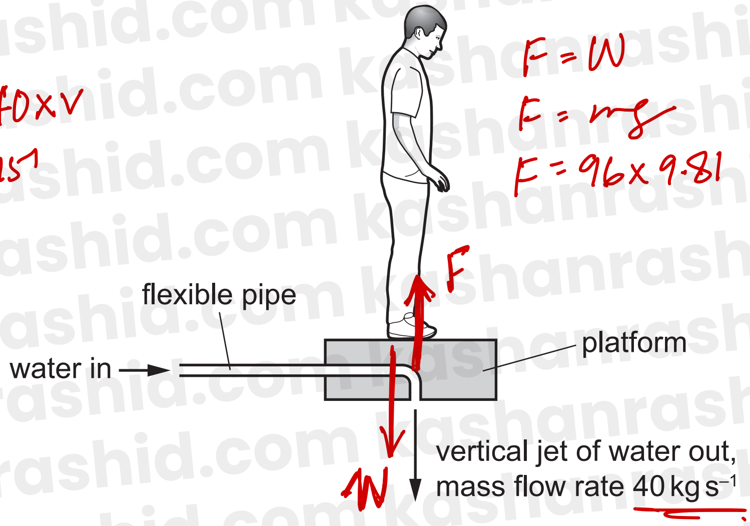
$96 \times 9.81 = 40 \times v$

$v = 24\ \text{ms}^{-1}$

$F = W$

$F = mg$

$F = 96 \times 9.81$



The resultant vertical force on the platform is zero. The combined mass of the man and platform is 96 kg. The mass of water that is discharged vertically downwards from the platform each second is 40 kg.

What is the speed of the water leaving the platform?

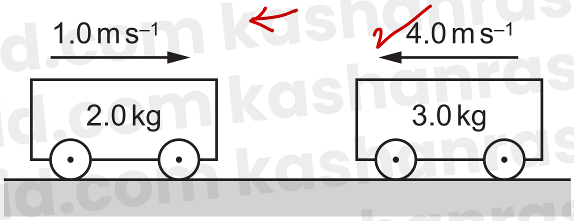
A $2.4\,\text{m}\,\text{s}^{-1}$ **B** $6.9\,\text{m}\,\text{s}^{-1}$ **C** $24\,\text{m}\,\text{s}^{-1}$ **D** $47\,\text{m}\,\text{s}^{-1}$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(2)(1) + (3)(-4) = (5)v$

$\frac{-10}{5} = v \qquad v = -2\,\text{m s}^{-1}$

11 Two frictionless trolleys are moving towards each other along the same horizontal straight line. Their masses and velocities are shown.



The trolleys collide and stick together.

What is the velocity of the trolleys after the collision?

A $2.0\,\text{m s}^{-1}$ to the left

B $2.0\,\text{m s}^{-1}$ to the right

C $2.8\,\text{m s}^{-1}$ to the left

D $2.8\,\text{m s}^{-1}$ to the right

$F = \frac{\Delta p}{\Delta t} = \frac{m(v-u)}{\Delta t} = \frac{0.008(0-300)}{100\times10^{-6}} = 24000\text{N}$

12 A bullet of mass 8.0 g travels at a speed of $300\,\text{m s}^{-1}$. The bullet hits a target and stops after a time of $100\,\mu\text{s}$.

What is the average force exerted by the target on the bullet?

A 24 N **B** 240 N **C** 2400 N **D** 24 000 N

3 (a) (i) State the principle of conservation of momentum.

For an isolated system, the total momentum of bodies before collision is equal to their total momentum after collision. [2]

(ii) State the difference between an elastic and an inelastic collision.

Energy is lost during inelastic collision. [1]

(b) An object A of mass 4.2 kg and horizontal velocity 3.6 m s^{-1} moves towards object B as shown in Fig. 3.1.

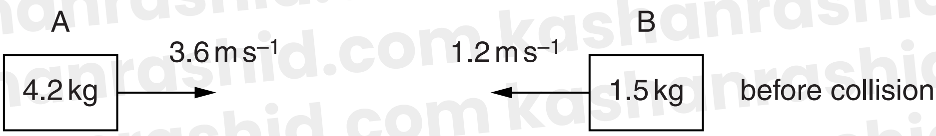


Fig. 3.1

Object B of mass 1.5 kg is moving with a horizontal velocity of 1.2 m s^{-1} towards object A.

The objects collide and then both move to the right, as shown in Fig. 3.2.

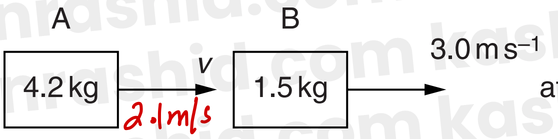


Fig. 3.2

Object A has velocity v and object B has velocity 3.0 m s^{-1}.

(i) Calculate the velocity v of object A after the collision.

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(4.2)(3.6) + (1.5)(-1.2) = (4.2)(v) + (1.5)(3)$

$v = 2.1 \text{ ms}^{-1}$

velocity = 2.1 m s^{-1} [3]

(ii) Determine whether the collision is elastic or inelastic.

approach = 3.6 + 1.2
= 4.8 ms^{-1}

unequal so inelastic

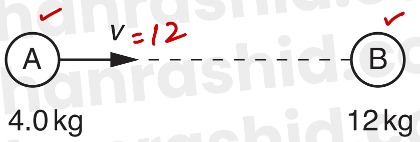
seperation = 3 − 2.1
= 0.9 ms^{-1}

[3]

3 **(a)** State the principle of conservation of momentum.

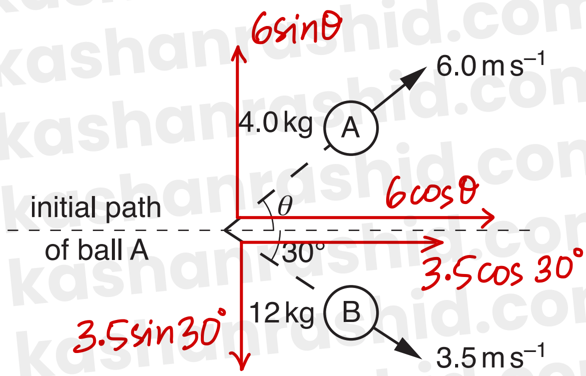
For an isolated system, the total momentum of bodies before collision is equal to their total momentum after collision [2]

(b) Ball A moves with speed v along a horizontal frictionless surface towards a stationary ball B, as shown in Fig. 3.1.



before collision

Fig. 3.1



after collision

Fig. 3.2 (not to scale)

Ball A has mass 4.0 kg and ball B has mass 12 kg.
The balls collide and then move apart as shown in Fig. 3.2.
Ball A has velocity 6.0 m s^{-1} at an angle of θ to the direction of its initial path.
Ball B has velocity 3.5 m s^{-1} at an angle of 30° to the direction of the initial path of ball A.

(i) By considering the components of momentum at right-angles to the direction of the initial path of ball A, calculate θ.

vertical axis

y-axis

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$0 + 0 = (4)(6\sin\theta) + (12)(-3.5\sin 30°)$$

$$0 = 24\sin\theta - 21$$

$$\frac{21}{24} = \sin\theta \qquad \theta = 61°$$

θ = 61 ° [3]

(ii) Use your answer in **(i)** to show that the initial speed v of ball A is $12\,m\,s^{-1}$.
Explain your working.

x-axis

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(4)(v) + 0 = (4)(6\cos 61) + (12)(3.5\cos 30°)$

$v = 12\,ms^{-1}$

[2]

(iii) By calculation of kinetic energies, state and explain whether the collision is elastic or inelastic.

before

$\frac{1}{2}(4)(12)^2 + 0$

$288\,J$

after

$\frac{1}{2}(4)(6)^2 + \frac{1}{2}(12)(3.5)^2$

$145.5\,J$

Inelastic

[3]

[Total: 10]