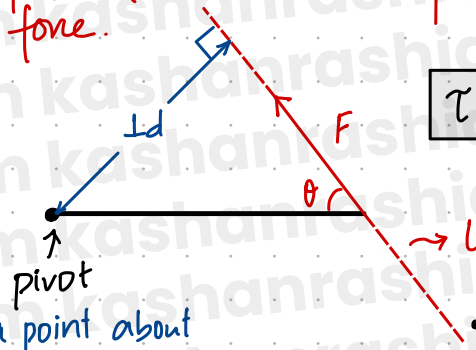
Moment of Force / Torque

It is the product of force and perpendicular distance from the pivot to the line of action of force.



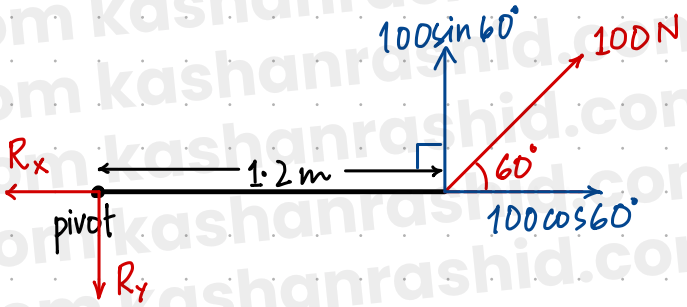
$$\tau = F \times \perp d$$

→ line of action

- Vector quantity
- SI Unit: Nm

(a point about which a body is allowed to rotate)

Example #1



a) Calculate the moment produced

* 100 cos60° doesnot produce any moment about pivot as it passes through the pivot. It doesnot have any perpendicular distance from it.

$\tau = F \times \perp d$ so $\tau = 100\sin 60° \times 1.2$

$\tau = 104$ Nm

b) Force exerted by pivot

R_x and R_y are reaction (contact) forces that act on the rod by the pivot and keeps the rod in translational equilibrium

$\Sigma F_x = 0$ so $R_x = 100\cos 60°$

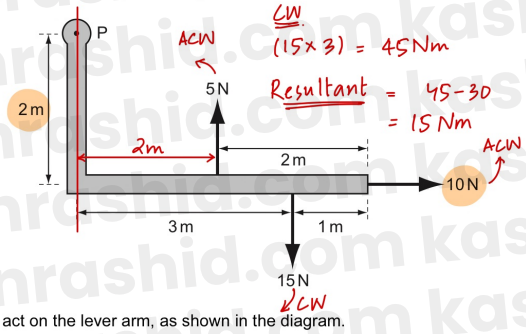
$\Sigma F_y = 0$ so $R_y = 100\sin 60°$

Resultant Moment

1

Additional Materials:

13 A rigid L-shaped lever arm is pivoted at point P.



ACW
(5 × 2) + (10 × 2) = 30 Nm

CW
(15 × 3) = 45 Nm

Resultant = 45 − 30
= 15 Nm
ACW

Three forces act on the lever arm, as shown in the diagram.

What is the magnitude of the resultant moment of these forces about point P?

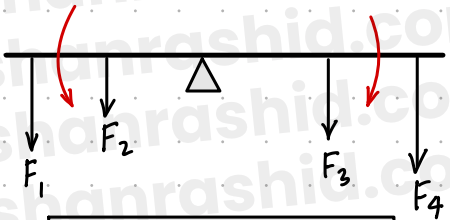
A 15 Nm B 20 Nm C 35 Nm D 75 Nm

☑ Horizontal force — Vertical distance

☑ Vertical force — Horizontal distance

Principle of Moments

For an object to be in equilibrium (rotational), sum of clockwise moments must be equal to sum of anticlockwise moments.



$$\Sigma \tau_{CW} = \Sigma \tau_{ACW}$$

* Center of Gravity

The point at which the whole weight of body appears to act.

For uniform bodies, the C.O.G is found at the geometric center i.e. at the middle of the body.

Example (DN/2015/P13)

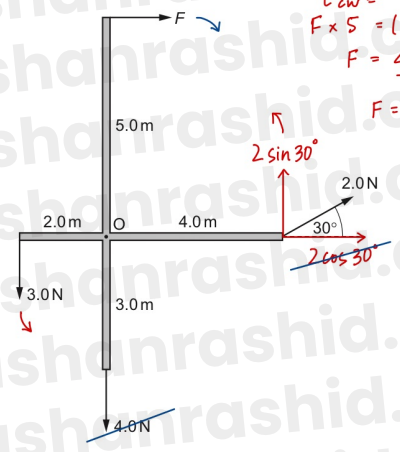
15 A cross-shaped structure, freely pivoted at O, has arms of lengths 5.0 m, 4.0 m, 3.0 m and 2.0 m. It is acted on by forces of 2.0 N, 3.0 N, 4.0 N and an unknown force *F*. The structure is in rotational equilibrium.

$\tau_{CW} = \tau_{ACW}$

$F \times 5 = (2\sin 30° \times 4) + (3 \times 2)$

$F = \frac{4+6}{5}$

$F = 2\text{N}$

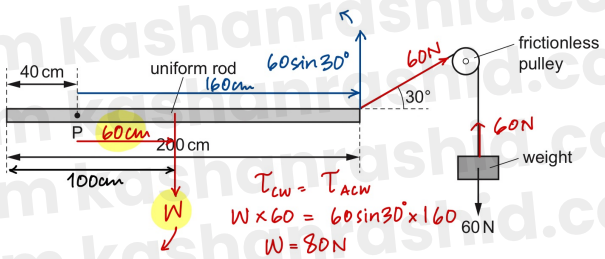


What is the magnitude of force *F*?

A 0.40 N **B** 2.0 N **C** 2.6 N **D** 4.4 N

Example DN/2018/P11

11 A uniform rod of length 200 cm is freely pivoted at point P. The rod is held horizontally in equilibrium by a 60 N weight that is attached to the rod by a string passing over a frictionless pulley.



$\tau_{CW} = \tau_{ACW}$

$W \times 60 = 60\sin 30° \times 160$

$W = 80\text{N}$

What is the weight of the rod?

A 30 N **B** 60 N **C** 80 N **D** 140 N

Example DN/2014/P11

12 A uniform ladder rests against a vertical wall where there is negligible friction. The bottom of the ladder rests on rough ground where there is friction. The top of the ladder is at a height *h* above the ground and the foot of the ladder is at a distance 2*a* from the wall.

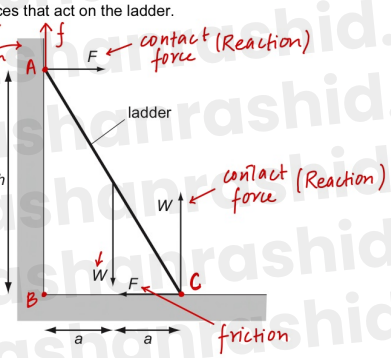
The diagram shows the forces that act on the ladder.

Point A as a pivot

$(W \times a) + (F \times h) = (W \times 2a)$

if wall had friction

contact force



Which equation is formed by taking moments?

A $Wa + Fh = 2Wa$

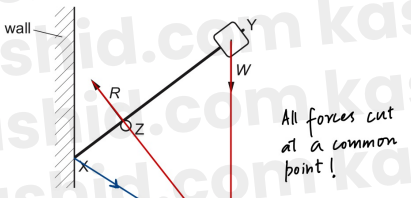
B $Fa + Wa = Fh$

C $Wa + 2Wa = Fh$

D $Wa - 2Wa = 2Fh$

When a body is in rotational equilibrium, all forces acting on that body intersect at a common point.

12 A light rigid rod XY has an object of weight *W* fixed at one end. The rod is in equilibrium, resting on a roller at Z and a vertical wall at X. The roller exerts a force *R* on the rod as shown. The diagram shows the directions, but not the magnitudes, of the forces *R* and *W*.



All forces cut at a common point!

What is the direction of the force on the rod at X?

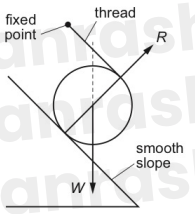
A **B**

D

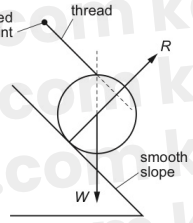
13 A cylinder of weight *W* is placed on a smooth slope. The contact force of the slope on the cylinder is *R*. A thread is attached to the surface of the cylinder. The other end of the thread is fixed.

Which diagram shows the cylinder in equilibrium?

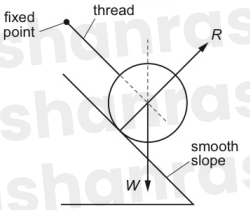
A



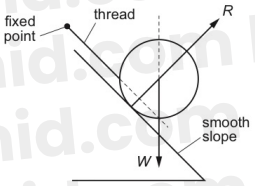
B



C

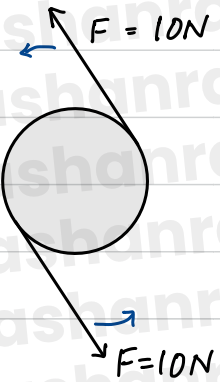


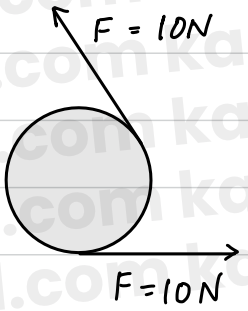
D



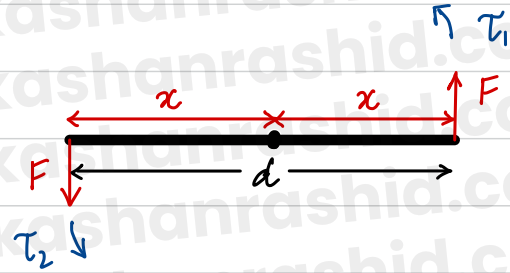
Torque of a couple

Couple forces are
- equal in magnitude
- opposite in direction
- acting at a distance from one another.





Not a pair of couple forces



$\tau = \tau_1 + \tau_2$

$= Fx + Fx$

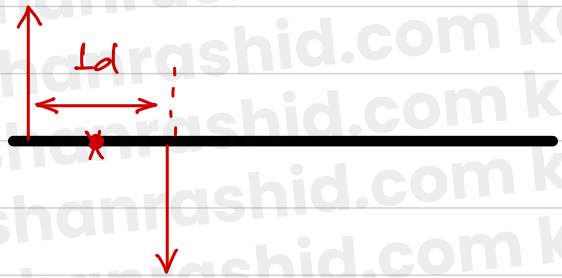
$\boxed{\tau = 2Fx}$

↳ torque of a couple.

$\tau = F \times 2x$

$\boxed{\tau = F \times d}$ → perpendicular distance between the two forces.

"It is the product of one of the two forces and the perpendicular distance between them."



DOUBLE PIVOT

8

3 A uniform plank AB of length 5.0 m and weight 200 N is placed across a stream, as shown in Fig. 3.1.

$F_A + F_B = 880 + 200$
$F_A + F_B = 1080$

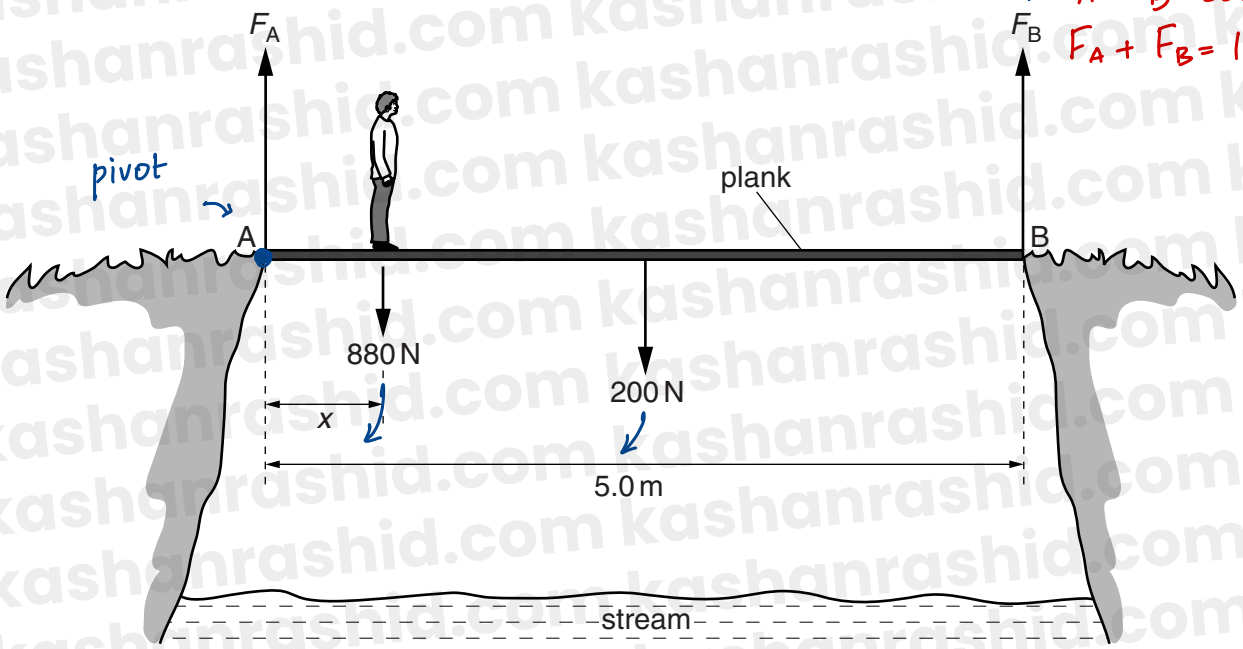


Fig. 3.1

A man of weight 880 N stands a distance x from end A. The ground exerts a vertical force F_A on the plank at end A and a vertical force F_B on the plank at end B.
As the man moves along the plank, the plank is always in equilibrium.

(a) (i) Explain why the sum of the forces F_A and F_B is constant no matter where the man stands on the plank.

For plank to be in equilibrium, total downward and upward force must be equal. As downward force is constant, so would the sum of $F_A + F_B$. [2]

(ii) The man stands a distance $x = 0.50$ m from end A. Use the principle of moments to calculate the magnitude of F_B.

"When taking any point as pivot, consider that point which helps reduce the unknown forces from moment equation."
"A" taken as pivot so that F_A doesn't become part of moment equation.

$T_{CW} = T_{ACW}$
$(880 \times 0.5) + (200 \times 2.5) = (F_B \times 5)$
$F_B = 188$ N

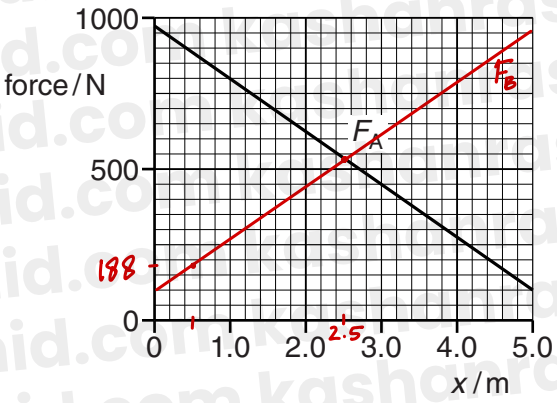
for F_A, $F_A + F_B = 1080$
$F_A + 188 = 1080$

$F_A = 892$ N

$F_B =$ ..... N [4]

9

(b) The variation with distance x of force F_A is shown in Fig. 3.2.



When $x = 2.5$m i.e. man stands in the middle, $F_A = F_B$.

Fig. 3.2

On the axes of Fig. 3.2, sketch a graph to show the variation with x of force F_B. [3]

3 A rod PQ is attached at P to a vertical wall, as shown in Fig. 3.1.

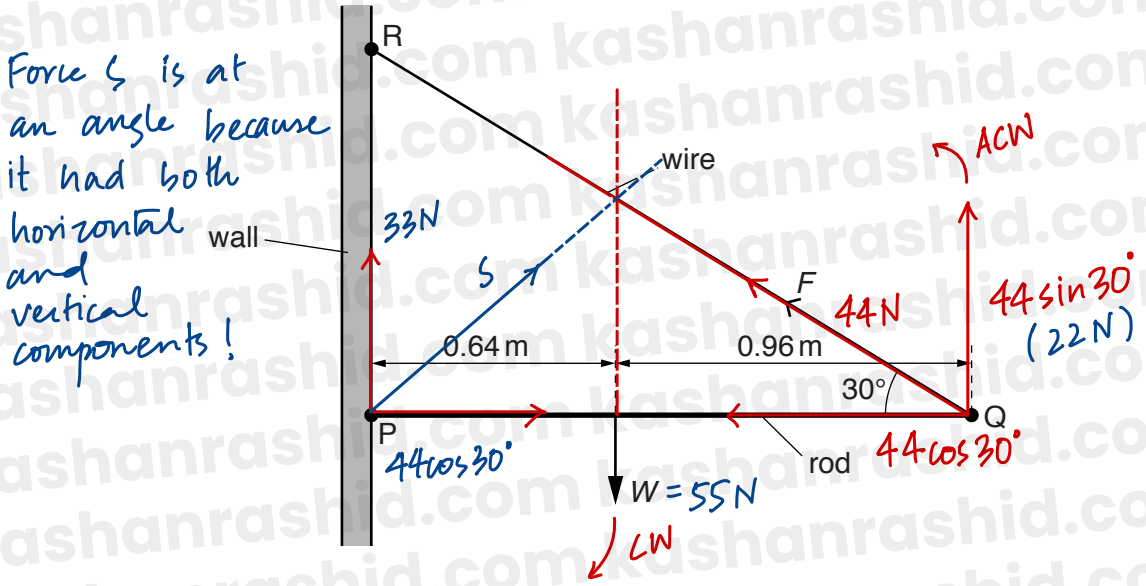


Fig. 3.1

The length of the rod is 1.60 m. The weight W of the rod acts 0.64 m from P. The rod is kept horizontal and in equilibrium by a wire attached to Q and to the wall at R. The wire provides a force F on the rod of 44 N at 30° to the horizontal.

(a) Determine

(i) the vertical component of F,

$44 \sin 30°$

vertical component = ... 22 ..... N [1]

(ii) the horizontal component of F.

$44 \cos 30°$
$(38.1\ N)$

horizontal component = ... 38 ..... N [1]

(b) By taking moments about P, determine the weight W of the rod.

$\tau_{CW} = \tau_{ACW}$
$W \times 0.64 = 22 \times 1.6$
$W = 55N$

W = ... N [2]

(c) Explain why the wall must exert a force on the rod at P.

Wall must exert a force balance $44\cos 30^\circ$ and the extra 33N of weight that is acting downwards.

[1]

(d) On Fig. 3.1, draw an arrow to represent the force acting on the rod at P. Label your arrow with the letter S. [1]